Range Assignment for High Connectivity in Wireless Ad Hoc Networks

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Abstract

Depending on whether bidirectional links or unidirectional links are used for communications, the network topology under a given range assignment is either an undirected graph referred to as the symmetric topology, or a directed graph referred to as the asymmetric topology. The Min-Power Symmetric (resp., Asymmetric) k-Node Connectivity problem seeks a range assignment of minimum total power subject to the constraint the induced symmetric (resp. asymmetric) topology is k-connected. Similarly, the Min-Power Symmetric (resp., Asymmetric) k-Edge Connectivity problem seeks a range assignment of minimum total power subject to the constraint the induced symmetric (resp., asymmetric) topology is k-edge connected.

The Min-Power Symmetric Biconnectivity problem and the Min-Power Symmetric Edge-Biconnectivity problem has been studied by Lloyd et. al [22]. They show that range assignment based the approximation algorithm of Khuller and Raghavachari [18], which we refer to as **Algorithm KR**, has an approximation ratio of at most $2(2-2/n)(2+1/n)$ for Min-Power Symmetric Biconnectivity, and range assignment based on the approximation algorithm of Khuller and Vishkin [19], which we refer to as **Algorithm KV**, has an approximation ratio of at most $8(1-1/n)$ for Min-Power Symmetric Edge-Biconnectivity.

In this paper, we first establish the NP-hardness of Min-Power Symmetric (Edge-)Biconnectivity. Then we show that **Algorithm KR** has an approximation ratio of at most 4 for both Min-Power Symmetric Biconnectivity and Min-Power Asymmetric Biconnectivity, and **Algorithm KV** has an approximation ratio of at most $2k$ for both Min-Power Symmetric k-Edge Connectivity and Min-Power Asymmetric k-Edge Connectivity. We also propose a new simple constant-approximation algorithm for both Min-Power Symmetric Biconnectivity and Min-Power Asymmetric Biconnectivity. This new algorithm is best suited for distributed implementation.

1 Introduction

Recently, range assignment problems for wireless ad hoc networks have been studied extensively. In wireless ad hoc networks no wired backbone infrastructure is installed and communication sessions are achieved either through a single-hop transmission if the communication parties are close enough, or through relaying by intermediate nodes otherwise. Omnidirectional antennas are used by all nodes to

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transmit and receive signals. Such antennas are attractive due to their broadcast nature. A single transmission by a node can be received by many nodes within its vicinity. We assume that every node can dynamically adjust its transmitting power based on the distance to the receiving node and the background noise. In the most common power-attenuation model [23], the signal power falls as $\frac{1}{d^\kappa}$ where d is the distance from the transmitter antenna and κ is a real *constant* between 2 and 5 dependent on the wireless environment. We assume that all receivers have the same threshold for signal detection, and normalize this threshold to one. With these assumptions, the power required to support a link between two nodes separated by a distance d is d^κ.

The network topology of a wireless ad hoc network, which consists of all possible one-hop communication links among the nodes, is determined by the transmission ranges of the nodes. Depending on whether *unidirectional* links or *bidirectional* links are used for communications, the network topology is represented by either a directed graph referred to as the *asymmetric topology*, or an undirected graph referred to as the *symmetric topology*. In the asymmetric topology, there is an arc from a node u to another node v if and only v is within the transmission range of u. In the symmetric topology, there is an edge between two nodes u and v if and only they are within the transmission ranges of each other. An example is depicted in Figure 1. Figure 1 (a) gives the positions and the transmission ranges of all nodes. The asymmetric topology and the symmetric topology are given in Figure 1 (a) and (b) respectively.

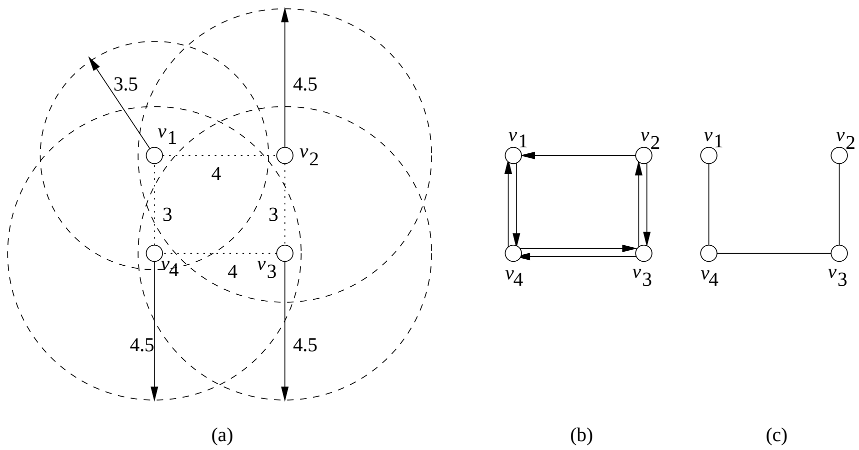

Figure 1: The network topology: (a) the nodes and their transmission ranges, (b) the asymmetric topology, and (c) symmetric topology.

Connectivity is one of the most important properties of an wireless ad hoc network. By asymmetric k-node (resp., k-edge) connectivity we mean the asymmetric topology is k-node (resp., k-edge) (strongly) connected, and by symmetric k-node (resp., k-edge) connectivity we mean the symmetric topology is k-node (resp., k-edge) connected. For $k = 1$, edge and node connectivity are identical to each other, and thus are simply referred to as connectivity. For $k = 2$, 2-node connectivity is simply referred to as biconnectivity, and 2-edge connectivity is simply referred to as edge-biconnectivity. With the same transmission ranges, the asymmetric connectivity is always not lower than the symmetric connectivity. If the transmission ranges are not identical, the asymmetric connectivity may be higher than the symmetric connectivity. Figure 2 shows an example in which the asymmetric topology is connected but the symmetric topology is disconnected. The network consists of nine nodes lying on a regular hexagon of side equal to one, with six nodes at the vertices of the hexagon and the other three nodes at the midpoints of three alternate sides of the hexagon. Three alternate nodes at the vertices have transmission range of one, and all others have the transmission range of one half. The asymmetric topology is connected, but the symmetric topology is not. On the other hand, if all nodes have the same transmission range, the asymmetric topology and the symmetric topology always have the same connectivity.

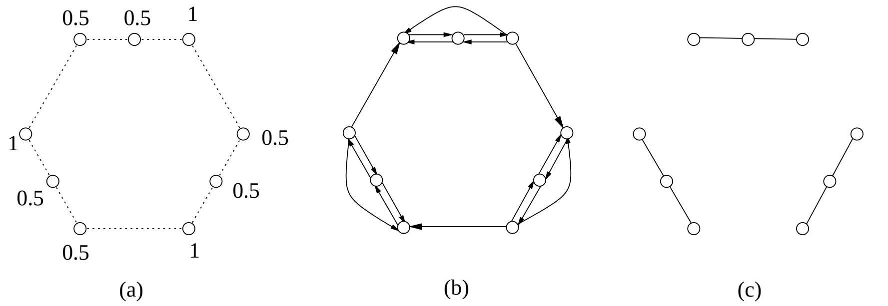

Figure 2: Asymmetric topology may have higher connectivity than symmetric topology. (a). The nodes lie in a regular hexagon of side equal to one, and their transmission ranges are given beside the nodes. (b) The asymmetric topology is connected. (c). The symmetric topology is disconnected.

The requirement on the network connectivity (either asymmetric or asymmetric) imposes a constraint on the transmission ranges of all nodes. A crucial issue is how to find a range assignment of the smallest total power to meet a specified connectivity requirement. The Min-Power Symmetric (resp., Asymmetric) k-Node Connectivity problem seeks a range assignment of minimum total power subject to the constraint the induced symmetric (resp. asymmetric) topology is k-connected. Similarly, the Min-Power Symmetric (resp., Asymmetric) k-Edge Connectivity problem seeks a range assignment of minimum total power subject to the constraint the induced symmetric (resp., asymmetric) topology is k-edge connected. Clearly, the smallest total power for asymmetric k-node (resp., edge) connectivity is no more than the smallest total power for symmetric k-node (resp., edge) connectivity.

The study of the Min-Power Asymmetric Connectivity problem was started by Chen and Huang [5], who gave a 2-approximation algorithm based on minimum spanning tree. Further contributions were made in [20] and [8]. The related broadcast problem was studied in [28], [26], and [6]. The recent survey [9] presents the state of the art for these "asymmetric" problems. The Min-Power Symmetric Connectivity problem was proposed in [2] and [4]. Both papers claim that Min-Power Symmetric Connectivity is NP-Hard, and [4] presents a $(1 + \ln 2)$-approximation algorithm. In the journal submission of [4], this approximation ratio is improved to 5/3.

The Min-Power Symmetric Biconnectivity problem has been first studied by Ramanathan and Rosales-Hain [24], which proposed one reasonable heuristic but without a proven approximation ratio. Lloyd et. al [22] studied both Min-Power Symmetric Biconnectivity and Min-Power Symmetric Edge-Biconnectivity. Among other results, they show that the range assignment based the approximation algorithm of Khuller and Raghavachari [18], which we refer to as **Algorithm KR**, has an approximation ratio of at most $2(2-2/n)(2+1/n)$ for Min-Power Symmetric Biconnectivity, and the range assignment based on the approximation algorithm of Khuller and Vishkin [19], which we refer to as **Algorithm KV**, has an approximation ratio of at most $8(1-1/n)$ for Min-Power Symmetric Edge-Biconnectivity.

In this paper, we present a reduction that establishes the NP-Hardness of both Min-Power Symmetric Two-Node Connectivity and Min-Power Symmetric Two-Edge-Connectivity. The NP-Hardness holds for plane instances, not only for arbitrary graph weights. We show that the range assignment based on the **Algorithm KR** has an approximation ratio of at most 4 for both Min-Power Symmetric Biconnectivity and Min-Power Asymmetric Biconnectivity. Specifically, we prove that the total power of this range assignment is less than four times the smallest power for asymmetric biconnectivity. We also show that the range assignment based on **Algorithm KV** has an approximation ratio of at most $2k$ for both Min-Power Symmetric k-Edge Connectivity and Min-Power Asymmetric k-Edge Connectivity. Specifically, we prove that the total power of this range assignment is less than $2k$ times the smallest power for asymmetric k-edge connectivity. As both algorithms are graph algorithms, the approximation ratios hold also if the nodes are in the three dimensional space, if the possible ranges come from a discrete set of values, if obstacles completely block the communication in between certain pairs of nodes, and if there is a maximum value on the ranges.

Although the range assignments based **Algorithm KR** and **Algorithm KV** have constant approximation ratios, they have very complicated implementations and are not practical for wireless ad hoc networks. This motivates us to seek a trade-off between the approximation ratio and the implementation complexity. We propose a very simple range assignment which achieves both symmetric and asymmetric biconnectivity. The total power of this range assignment is less than 8 for $\kappa = 2$, or $3.2 \cdot 2^{\kappa}$ for $\kappa > 2$ times the smallest power for asymmetric connectivity for plane instances.

The remaining of this paper is organized as follows. Due to space limitations, we present the NP-hardness of Min-Power Symmetric (Edge-) Biconnectivity in Appendix A. In Section 2, we introduce related graph-theoretic results and some terms and notations. In Section 3 and Section 4, we derive tighter upper bounds on the approximation ratios of the range assignments based **Algorithm KR** and **Algorithm KV** respectively. In Section 5, we present the new algorithm, MST-Augmentation, and analyze its approximation ratio. Finally, in Section 6, we conclude the paper and report preliminary experimental results.

2 Preliminaries

A directed graph $D = (V, A)$ is said to be a *branching* (or arborescence) rooted at some vertex $s \in V$ if $|A| = |V| - 1$ and there is a path to s from any other vertex. In other words, branchings in directed graphs are a directed analog to spanning trees in undirected graphs.

Theorem 1 (Edmonds) *[11] Suppose that, given a directed graph $D = (V, A)$ and a specified vertex $s \in V$, there are k arc-disjoint paths to s from any other vertex of D. Then D has k arc-disjoint branchings rooted at s.*

Theorem 2 (Whitty) *[27] Suppose that, given a directed graph $D = (V, A)$ and a specified vertex $s \in V$, there are two internally vertex-disjoint paths to s from any other vertex of D. Then D has two arc-disjoint branchings rooted at s such that for any vertex $v \in V - s$ the two paths to s from v uniquely determined by the branchings are internally vertex-disjoint.*

Consider a directed graph $D = (V, A)$, a specified vertex $s \in V$, and a positive integer k. The cheapest subgraph of D that has k arc-disjoint paths to s from every other vertex, if there is any, must be the union of k arc-disjoint branchings rooted at s and can be found in polynomial time by the weighted matroid intersection algorithm due to Lawler [21] and Edmonds [12]. The fastest implementation of a weighted matroid intersection algorithm is given by Gabow [14]. Given a vertex $r \in V$, the cheapest subgraph of D that has k internally vertex-disjoint paths to r from every other vertex, if there is any, can also be found in polynomial time by an algorithm due to Frank and Tardos [13], or a faster algorithm due to Gabow [15].

We will also make use of a corollary of Menger's Theorem, the so-called Fan Lemma.

Theorem 3 (Fan Lemma) *[10] Suppose that D is a k-vertex connected directed graph and U is a proper subset of its vertices with $|U| = k$. Then for any vertex v not in U, there are k internally vertex-disjoint paths that link v to distinct vertices of U.*

The bidirected version of an undirected graph G is a directed graph obtained by replacing every edge of G with two oppositely oriented arcs. The undirected version of a directed graph D is an undirected graph obtained by ignoring the directions of the arcs of D.

From now on, we model the wireless ad hoc network by a weighted complete graph $G = (V, E, c)$ with $c(e) = \|e\|^{\kappa}$ where $\|e\|$ is the length of the edge e. Every range assignment is specified by a spanning graph H as follows. The transmission power of node v with respect to H, denoted by $p_H(v)$, is defined by

$$p_H(v) = \max_{u \in N_H(v)} c(vu).$$

Clearly, the symmetric topology induced by this range assignment contains H as a subgraph, and the asymmetric topology induced by this assignment contains the bidirected version of H as a subgraph. Thus, the range assignment specified by H achieves at least the connectivity of H.

For any spanning subgraph H of G, we define the power cost of H as

$$p(H) = \sum_{v \in V(H)} p_H(v).$$

Then $p(H)$ is exactly the total power of the range assignment induced by H. We also define the weight of H as

$$c(H) = \sum_{e \in E(H)} c(e).$$

The two parameters $p(H)$ and $c(H)$ are related by the following previously known lemma.

Lemma 4 *For any spanning subgraph H of G, $p(H) \leq 2c(H)$.*

Proof. Let H be a subgraph of G. Then,

$$\begin{aligned} p(H) &= \sum_{v \in V} p_H(v) = \sum_{v \in V} \max_{u \in N_H(v)} c(vu) \\ &\leq \sum_{v \in V} \sum_{u \in N_H(v)} c(vu) = 2 \sum_{e \in E(H)} c(e) = 2c(H). \end{aligned}$$

■

For directed spanning subgraphs Q, we define similarly $p_Q(v) = max_{vu \in Q} c(cu)$ for every vertex v, and $p(Q) = \sum_{v \in V} p_Q(v)$.

3 Algorithm KR for k-Edge Connectivity

Algorithm KR [18] constructs a k-edge connected spanning subgraph H as follows. For some node s, let D_s be the minimum-weight directed subgraph of the bidirected version of G in which there are k arc-disjoint paths to s from every other vertex in V. Let H be the undirected version of D_s for an arbitrary node s. Then, as shown in [18], H is k-edge connected.

Let *opt* be the power cost of an optimum range assignment for asymmetric k-edge connectivity. We have the following theorem.

Theorem 5 $p(H) \leq 2k \cdot opt$.

Proof. Consider Q, the directed graph given by the optimum range assignment. Q is strongly k-edge connected, and therefore by Theorem 1 Q contains k arc-disjoint branchings rooted at s: $T_1, T_2, \cdots, T_k$.

As $\cup_{i=1}^k T_i$ is a feasible solution solution for the directed subgraph computed by the algorithm, $c(D_s) \leq \sum_{i=1}^k c(T_i)$. For any vertex v and $1 \leq i \leq k$, denote by $a_i(v)$ the parent of v in $T_i(v)$. Given v, $p_Q(v) = \max_{vu \in Q} c(uv) \geq \max_{1 \leq i \leq k} c(va_i(v)) \geq \frac{1}{k} \sum_{1 \leq i \leq k} c(va_i(v))$, and therefore $opt = p(Q) \geq \frac{1}{k} \sum_{1 \leq i \leq k} c(T_i)$. Using Lemma 4, we conclude:

$$p(H) \leq 2c(H) \leq 2c(D_s) \leq 2 \sum_{i=1}^{k} c(T_i) \leq 2k \cdot opt$$

■

Theorem 5 implies that the approximation ratio of **Algorithm KR** is at most $2k$.

4 Algorithm KV for Biconnectivity

Algorithm KV [19] constructs a 2-node connected spanning subgraph H as follows.

1. Let xy be the edge of G of minimum weight and s an vertex not in V. Construct weighted directed graph D as follows: Replace every edge of G with two oppositely-oriented arcs of the same weight and then add two arcs xs and ys of weight 0.

2. Let D' be the minimum-weighted subgraph of D in which there are two internally vertex-disjoint directed paths to s from every vertex in V. (D' can be obtained by using the algorithm of Frank and Tardos [13], or a faster algorithm by Gabow [15]).

3. Output the subgraph H of G which contains the edge xy and every edge of G with at least one of its two directed copies in D'.

As shown in [19] (or in [17], pages 246-247), H is two-connected. Let *opt* be the power cost of an optimum range assignment for asymmetric 2-node connectivity. We have the following theorem.

Theorem 6 $p(H) \leq 4 \cdot opt$.

Proof. Consider Q, the directed graph given by the optimum range assignment, to which we add the arcs xs and ys of weight 0. Using Theorem 3 (Fan Lemma), for any vertex v other than x and y, Q has two internally vertex-disjoint directed paths that link v to x and y respectively. Therefore, in Q, every vertex v has two internally vertex-disjoint directed paths linking it to s. Using Theorem 2, Q has two arc-disjoint branchings rooted at s: A_1 and A_2 such that, for every vertex $v \in V$, the two paths in A_1 and A_2 from v to r are internally vertex-disjoint.

As $A_1 \cup A_2$ is a feasible solution for the directed subgraph we needed in step 2 , $c(D') \leq c(A_1) + c(A_2)$. For any vertex v and $1 \leq i \leq 2$, denote by $a_i(v)$ the parent of v in $A_i(v)$. Given v, $p_Q(v) = \max_{vu \in Q} c(uv) \geq (c(va_1(v)) + c(va_2(v)))/2$, and therefore $opt = P(Q) \geq (c(A_1) + c(A_2))/2$.

Using Lemma 4, we conclude:

$$p(H) \leq 2c(H) = 2c(D') \leq 2(c(A_1) + c(A_2)) \leq 4opt$$

■

Theorem 6 implies that the approximation ratio of **Algorithm KR** is at most 4.

5 Algorithm MST-Augmentation for Biconnectivity

In this section, we present a simple algorithm which produces a biconnected spanning graph H by augmenting an MST. The algorithm first finds an Euclidean MST T and initializes H to T. At any non-leaf node v of T, a local Euclidean MST T_v over all the neighbors of v in T is constructed and added to H. Thus the H is a union of the big MST T and many small MSTs. H is 2-connected, as it follows from the following argument. Only internal nodes of T can be articulation points; let u be such a node. Removing u from T creates a number of connected components of T, each having one vertex neighbor with u in T. But the neighbors of u in T remain connected by T_u, the local MST which does not include u.

We refer to this algorithm as **MST-Augmentation**. Besides being simple and very fast (as every vertex has constant degree in T, total running time is dominated by constructing T and is $O(n \log n)$), this algorithm is best suited to efficient distributed implementation. Another advantage of this algorithm is the independence of the path-loss exponent.

To bound the approximation ratio of **MST-Augmentation**, we introduce a geometric constant α defined below. Let o be the origin of the Euclidean plane. A set U of at least two points is called as a *star-set* if its Euclidean MST for $\{o\} \cup U$ is a star centered at o. The star is denoted by S_U. Note that each star-set contains at least two but at most six points. For any star-set U, let T_U be the minimum spanning tree of U. Then α is defined as the supreme of the ratio $c(T_U)/c(S_U)$ over all star-sets.

Lemma 7 *For any $\kappa \geq 2$, $2^{\kappa-1} \leq \alpha \leq 1.6 \cdot 2^{\kappa-1}$. If $\kappa = 2$, then $\alpha = 2$.*

Due to space limitations, the proof of this lemma appear in the Appendix B.

Now we are ready to represent the upper bound on $p(H)$ in terms of α and the power cost of an optimum range assignment for asymmetric connectivity which is denoted by *opt*.

Theorem 8 *$p(H) < 4\alpha \cdot opt$.*

The proof of this theorem consists of the following several lemmas. The next lemma is implicit in previous work and it follows immediatly from the fact that T is a minimum spanning trees and one argument used in the proof of Theorem 5.

Lemma 9 *$c(T) < opt$.*

Let E_1 be the set of all edges of T incident to leaves. Let E_2 be the set of all edges of the trees T_v for all non-leaf nodes v. Let H' be the graph $(V, E_1 \cup E_2)$. Then H' is a subgraph of H, and thus $p(H) \geq p(H')$. The next lemma states that the equality actually holds.

Lemma 10 *For every node v, $p_H(v) = p_{H'}(v)$, and consequently $p(H) = p(H')$.*

Proof. We prove the lemma by contradiction. Assume that $p_H(v) > p_{H'}(v)$ for some node v. Let $p_H(v) = c(uv)$. Then uv must be an edge of T and neither of u and v is a leaf. Since u is not a leaf, u has an neighbor w other than v such that vw is an edge in T_u. So vw is an edge of E_2. Since both uv and uw are edges of the MST, $\|uv\| \leq \|wv\|$, and thus $c(uv) \leq c(wv)$. Therefore,

$$p_H(v) = c(uv) \leq c(wv) \leq p_{H'}(v),$$

which is a contradiction. ■

The next lemma provides an upper bound in the total weight of H'.

Lemma 11 $c(H') \leq 2\alpha \cdot c(T)$.

Proof. From Lemma 7, we have $c(T_u) \leq \alpha \sum_{uv \in T} c(uv)$. Then $c(H') = c(E_1) + c(E_2) = \sum_{u \text{ leaf}} \sum_{vu \in T} c(uv) + \sum_{u \text{ internal}} c(T_u) \leq \alpha \sum_{u \text{ leaf}} \sum_{vu \in T} c(uv) + \alpha \sum_{u \text{ internal}} \sum_{vu \in T} c(uv) = 2\alpha c(T)$, as every edge of T appears exactly twice in the summation. ■

Now Theorem 8 follows immediately from Lemma 4, Lemma 9, Lemma 10, and Lemma 11:

$$p(H) = p(H') \leq 2c(H') < 4\alpha \cdot c(T) < 4\alpha \cdot opt.$$

Theorem 8 and Lemma 7 imply that the approximation ratio of **MST-Augmentation** is at most 8 for $\kappa = 2$ and at most $3.2 \cdot 2^{\kappa}$ for general κ.

6 Conclusion

We presented improved analysis for existing algorithms for Min-Power Symmetric Biconnectivity and Min-Power Symmetric k-Edge Connectivity, and showed the symmetric output of these algorithms is also a good approximation for Min-Power Asymmetric Biconnectivity and Min-Power Asymmetric k-Edge Connectivity, respectively. We showed that Min-Power Symmetric Biconnectivity and Min-Power Symmetric Edge-Biconnectivity is NP-Hard. We introduced the new algorithm **MST-Augmentation** and showed it also has constant approximation ratio.

We are aware of instances where the min-power asymmetric two-connected topology uses only 7/10 of the min-power symmetric two-connected topology. It would be interesting to find how small this ratio could be. By our analysis of the Min-Power Biconnectivity **Algorithm KR**, the ratio is at least 1/4, and in fact we can show the ratio is at least 1/3. By comparison, the ratio of min-power symmetric connected topology to min-power asymmetric connected topology is known to be at least 1/2, and this bound is tight (see for example the journal version of [4]).

Preliminary experimental results for Min-Power Symmetric Biconnectivity show that on random instances with 100 nodes, the following hold:

- "smart" local optimization algorithms improve by an average of 6% the Ramanathan and Rosales-Hain algorithm, with a maximum improvement of 18%. The Ramanathan and Rosales-Hain algorithm has a local optimization phase and on average uses 29% less power than **MST-Augmentation**.

- Our best heuristics have power 75% to 250% more than the cost of the minimum spanning tree (the only easily computable lower bound for the problems). The average power used is 110% more than the cost of the minimum spanning tree.

- For our best algorithms, the power required to ensure Symmetric Biconnectivity is on average 61.6% higher than the power required for Symmetric Connectivity. Our heuristics for Symmetric Connectivity are very good [1], but we still do not know the quality of the Symmetric Biconnectivity solutions our heuristics produce. Note that the minimum power for Symmetric Biconnectivity could be higher than the minimum power for Symmetric Connectivity by a factor of 2^{κ}, as shown by an example of n nodes being equidistant on a line.

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Appendix A

In this section we describe the reduction proving the NP-Hardness of both Min-Power Symmetric Biconnectivity and Min-Power Symmetric Edge-Biconnectivity. NP-Hardness holds for plane instances, not only for arbitrary graph weights. The reduction is from Hamiltonian Circuit in Planar Cubic Graphs, proved to be NP-Complete in [16].

Let $G = (V, E)$ be a planar cubic (all vertices having degree three) graph with n vertices. We construct an instance U of Min-Power Symmetric Two-(Edge)-Connectivity as follows. We first apply the polynomial time algorithm in [3] to obtain a planar orthogonal grid drawing of G in which each vertex u has integer coordinates, each edge uv has at most one bend, and each horizontal or vertical line segment has length between 6 and a polynomial function of n. Scale the construction up by n, so that a point x on the embedding of edge uv with $\|xu\| > n$ and $\|xv\| > n$ is at distance at least n to any point on some embedded edge other than uv. Let L be the total length of the edges. Then L is bounded by a polynomial in n.

Next, subdivide every edge of length l into lL^2 equidistant points but remove in the middle of the edge, in a place not containing a bend, L^2 of these new points, leaving a *gap* of length 1. Place a node in each of the points mentioned above. Finally, for every already placed node in the plane, place arbitrarily at distance $1/L^2$ to it another new node; two such nodes are called *twins*. The total number of nodes introduced is $O(L^3)$, and therefore the construction is polynomial.

If we consider the graph induced only by nodes at most $3/L^2$ apart, it has n components, each corresponding to a vertex of the original graph G. We call such a component *the cluster* of the original vertex v. Moreover, each component is two connected since each node has a twin, and the distance in between the twins of two "close" (at distance $1/L^2$) nodes is at most $3/L^2$. For an illustration of one cluster, see Figure 3.

The instance so constructed has n' nodes. Recall that the power of a node is at least the square of its assigned range. If the original graph is Hamiltonian, we obtain a range assignment of total power not exceeding $2n + 9n'/L^4$ by assigning to every node a range of $3/L^2$ and, for every edge uv of the

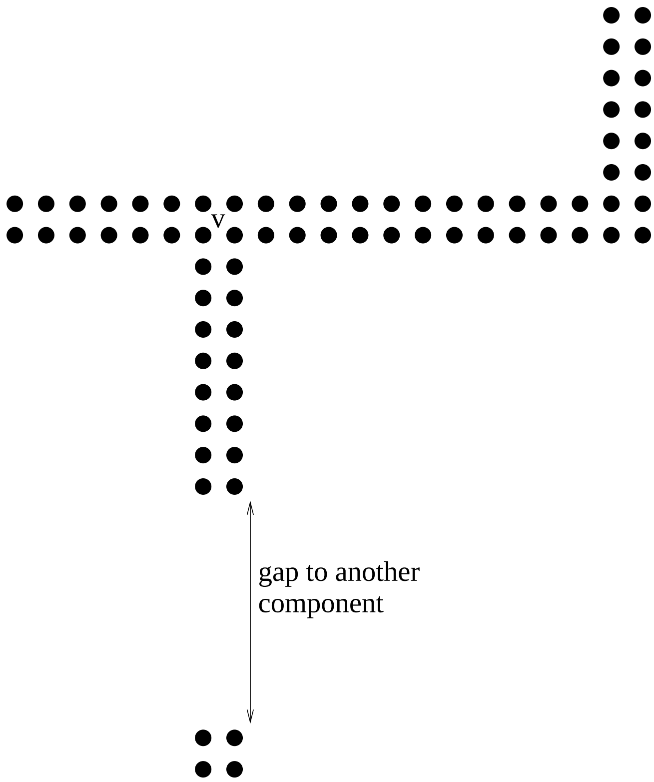

Figure 3: The cluster of v, with a gap of length 1 to another cluster

Hamiltonian path, we pick the two nodes next to the uv-gap, one in the cluster of u and one in the cluster of v, and assign them range 1. Note that $n' \leq 2L^3$ and therefore $2n + 3n'/L^4 < 2n + 1$.

Next we show that any range assignment of total power less than $2n+1$ implies that the original graph G is Hamiltonian. Let H' be the two-(edge-)connected established by the range assignment, and H be the multigraph obtained from H' by contracting every cluster to a single vertex. Every cluster must be incident to at least two edges of H. For a node x in a cluster to have edges of H' incident to nodes in two other clusters, it must have a range of at least n, contributing at least n^2 to the total power. So we may assume that any node is, in H', incident only with nodes in its own cluster, or only one extra cluster. A range of at least 1 is needed to establish links to another cluster.

For $Q \subseteq V$, let $P(Q)$ be the minimum total power required to establish the edges of H' with both endpoints in the clusters of $H[Q]$, the subgraph of H induced by Q. We have:

Claim 12 *If $Q \subseteq V$, $|Q| \geq 3$, and $H[Q]$ is edge-biconnected, then $P(Q) \geq 2|Q|$.*

Proof. Indeed, if every cluster corresponding to Q has two vertices with range 1, then the claim holds. If the cluster corresponding to a vertex $v \in Q$ has only one node x with range at least 1, then v is adjacent in $H[Q]$ to only one other vertex, which we call u, by at least two parallel edges. Then, in the cluster of u, two nodes must have range at least 1 and be adjacent to x in H'. Also, $H[Q-v]$ must be two-edge connected. If $|Q-v| = 2$, the same reasoning as above implies that $P(Q-v) \geq 3$ and therefore $P(Q) \geq 6$. If $|Q| \geq 4$, the result follows by induction. ■

The previous claim and its proof imply that if $H[Q]$ is two edge connected, $|Q| \geq 4$, and $P(Q) < 2|Q|+1$, then every cluster corresponding to Q has exactly two nodes with range at least 1, establishing links to two other clusters. For $Q = V$, this implies that $H[V]$ is Hamiltonian, and therefore G is Hamiltonian. In conclusion, we have:

Theorem 13 *Min-Power Symmetric Biconnectivity and Min-Power Symmetric Edge-Biconnectivity are NP-Hard.*

Appendix B

In this section we present the proof of Lemma 7, which states that for any $\kappa \geq 2$, $2^{\kappa-1} \leq \alpha \leq 1.6 \cdot 2^{\kappa-1}$, and that if $\kappa = 2$, then $\alpha = 2$.

Proof. The lower bound $2^{\kappa-1}$ is achieved by U consisting of two points u_1 and u_2 such that o is the midpoint of the line segment u_1u_2. Next, we prove the upper bound $1.6 \cdot 2^{\kappa-1}$. Consider any star-set U. If U has exactly six points, then these points form a regular hexagon centered at o, and hence

$$c\left(T_U\right) = \frac{5}{6} c\left(S_U\right) < 1.6 \cdot 2^{\kappa-1} c\left(S_U\right).$$

So we assume U has $m \leq 5$ points. For any two points u and w in U,

$$\begin{aligned} c(uw) &= \|uw\|^\kappa \leq (\|ou\| + \|ow\|)^\kappa \\ &= 2^\kappa \left(\frac{\|ou\| + \|ow\|}{2} \right)^\kappa \\ &\leq 2^\kappa \frac{\|ou\|^\kappa + \|ow\|^\kappa}{2} \\ &= 2^{\kappa-1} (c(ou) + c(ow)). \end{aligned}$$

Thus, the total weight of the convex polygon formed by the points of U is at most $2^\kappa c(S_U)$. On the other hand, as removing the largest edge of the polygon creates a tree on U, $c(T_U)$ is at most $\left(1 - \frac{1}{m}\right)$ times the total weight of this polygon. Thus,

$$c(T_U) \leq \left(1 - \frac{1}{m}\right) \cdot 2^\kappa c(S_U) \leq \left(1 - \frac{1}{5}\right) \cdot 2^\kappa c(S_U) = 1.6 \cdot 2^{\kappa-1} c(S_U).$$

The lemma thereby follows

Now we assume $\kappa = 2$ and show that $\alpha = 2$. Since $\alpha \geq 2$, we only have to show that $\alpha \leq 2$. Consider a star-set

$$U = \{(a_i, a_i) : 1 \leq i \leq m\}.$$

Let K_U denote the complete graph over U. We first claim that

$$c(S_U) \geq \frac{1}{m} c(K_U).$$

To see this, we make use of the following inequality.

$$\begin{aligned} \sum_{i=1}^m a_i^2 &= \frac{\left(\sum_{i=1}^m a_i\right)^2 + \sum_{1 \leq i < j \leq m} (a_i - a_j)^2}{m} \\ &\geq \frac{\sum_{1 \leq i < j \leq m} (a_i - a_j)^2}{m}. \end{aligned}$$

Thus,

$$\begin{aligned} c(S_U) &= \sum_{i=1}^m \left(a_i^2 + b_i^2\right) \\ &\geq \frac{\sum_{1 \leq i < j \leq m} \left[(a_i - a_j)^2 + (b_i - b_j)^2\right]}{m} \\ &= \frac{1}{m} c(K_U). \end{aligned}$$

Next, we claim that

$$c(T_U) \leq \frac{2}{m} c(K_U).$$

This claim can be proved by a simple counting argument. Note that a complete graph of order m has m^{m-2} spanning trees, and each edge appears in

$$\frac{m^{m-2}(m-1)}{\frac{m(m-1)}{2}} = 2m^{m-3}$$

spanning trees (see, for example, Chapter 2 of [25]). The total weight of all spanning trees of K_U is thus $2m^{m-3}c(K_U)$. Hence,

$$c(T_U) \leq \frac{2m^{m-3}c(K_U)}{m^{m-2}} = \frac{2}{m}c(K_U).$$

From the two previous claims, we have

$$\frac{c(T_U)}{c(S_U)} \leq \frac{\frac{2}{m}c(K_U)}{\frac{1}{m}c(K_U)} = 2.$$

So the lemma follows for $\kappa = 2$. ■