

St. Ambrose University
Department of Mathematics
Math. 192 Final Exam

Name: _____

Solve each of the following problems SHOWING ALL DETAILS . Unsupported or unclear answers will not receive partial credit. (7 pts each)

1)

a) Prove the following reduction formula

$$\int \tan^n x dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x dx.$$

b) Using integration by parts, evaluate $\int \frac{xe^x}{(x+1)^2} dx$.

2)

a) Evaluate $\int \frac{2x+3}{4x^2+4x+5} dx$

b) Evaluate $\int \frac{e^t}{e^{2t}-4} dt$ (Hint: Make a substitution and change the integrand to a rational function, then use partial fractions).

3)

a) Evaluate $\int \frac{x dx}{\sqrt{9 - x^2}}$

b) Use L'Hopital's rule to evaluate $\int_1^{+\infty} \frac{\ln x}{x^2} dx$.

4)

a) Solve the following DE by any method

$$y' + y = \cos(e^x)$$

5)

a) Show that the following sequence is strictly increasing or strictly decreasing.

$$a_n = \{\tan^{-1} n\}_{n=1}^{\infty}$$

b) Find the general term of the sequence starting with $n = 1$, determine if the sequence converges, and if so find the limit.

$$a_n = -1, 2, -3, 4, -5, \dots$$

6)

a) Determine if the following series converges and if so find its sum.

$$\sum_{k=1}^{\infty} \frac{1}{9k^2 + 3k - 2}$$

b) $\sum_{k=1}^{\infty} 5^k 7^{1-k}$

7) Express the decimal $13.\overline{6666}$... as a fraction using geometric series.

8) Use any of the convergence tests to determine whether the following series converges.

$$\sum_{k=1}^{\infty} \left(1 + \frac{1}{k}\right)^{-k}$$

9) Use the integral test to determine if the following series converges or diverges

$$\sum_{k=1}^{\infty} \frac{k}{1+k^2}$$

10) Find the Taylor series for the given functions about the given point.

a) $f(x) = e^{-x}$, $x_0 = 1$

b) $f(x) = \cos x$, $x_0 = \frac{\pi}{2}$

11) Given that $f(x, y) = y^3 e^{-5x}$. find $f_{yyxx}(0, 1)$